ACTEX Learning

Study Manual for Exam 9

13th Edition

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A CAS Exam

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NOTES

I have been updating this manual since 2015. There have been enough syllabus changes and new problems that the majority of the outline material is mine. For the current update, I reflected the 2023 syllabus material. In particular, I revised the manual to reflect the 12th edition of the Bodie, Z.; Kane, A.; and Marcus, A.J. *Investments* textbook. Effective with the 2020 exams, the Casualty Actuarial Society is no longer providing questions and answers for its exams, so the questions and answers from the 2020 and subsequent exams are not included in this update. I did add some questions and answers for the 2023 edition.

When readings have been taken off the syllabus, I retained those questions from past exams that I think are relevant to the current readings, making changes to reflect the then-new syllabus material where appropriate. There is a section at the end of the Study Guide for questions that apply to multiple readings that appeared on the 2018 and 2019 exams as the first two problems on each exam. As a practical matter, I suggest completing all other questions before answering questions that require material from multiple readings, regardless of where questions are placed in the exam.

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for CAS exams. Students may also request past exams directly from the CAS or find them on the CAS website. I am very grateful to the CAS for its cooperation and permission to use this material. The CAS is not responsible for the structure or accuracy of this manual. In some cases, questions and answers have been edited or altered to be more accurate, reflect syllabus changes, or provide a better organized manual. Students should keep in mind that there may be more than one correct way to answer a question even if only one is shown.

Exam questions are identified by numbers in parentheses at the end of each question. Questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned. MC indicates that a multiple choice question has been converted into a true/false question. MTS in place of the second number indicates questions I added, with the first number denoting the year in which they were added.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted.

My thanks to Peter J. Murzda, Jr., FCAS, ASA, who originally wrote this manual, and Chris Van Kooten, FCAS, FCIA, who previously updated it.

I made a conscientious effort to eliminate mistakes and incorrect answers, but a few may remain. I am grateful to students who previously pointed out errors and encourage those who find others to bring them to my attention. If you find any errors, please submit them to support@actexmadriver.com. Please check the ACTEX Learning website for corrections subsequent to publication.

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January 2023

Bodie Kane Marcus

Investments, Twelfth Edition: Chapter 6
Risk Aversion and Capital Allocation of Risky Assets

OUTLINE

I. Notation

A. $R(r)$ Expected return
B. σ^2 Variance of returns
C. U Utility value
D. A Index of investor's risk aversion
E. P Portfolio of risky assets
F. r_P Risky rate of return of P
G. $E(r_P)$ Expected rate of return of P
H. σ_P Standard deviation of P
I. F Risk-free asset
J. r_f Risk-free return
K. r_f^B Rate charged to borrow
L. E Proportion of risky portfolio in equities
M. B Proportion of risky portfolio in bonds
N. C Complete portfolio
O. r_C Rate of return of C
P. y Proportion of C that is made up of the risky portfolio
Q. y^* Optimal allocation to the risky portfolio
R. CAL Capital allocation line
S. S Reward to volatility ratio = slope of the CAL

II. Risk and Risk Aversion

A. Risk, Speculation and Gambling

1. Speculation – assumption of considerable investment risk to obtain commensurate gain

   a. Considerable risk – risk sufficient to affect decision
   b. Commensurate gain – positive risk premium

2. Gamble – bet or wager on an uncertain outcome (no positive risk premium)

3. Fair game – risky investment with zero risk premium

4. Heterogeneous expectations – different beliefs in probability of a positive return

5. Even highly risky positions may be assumed willingly by risk-averse investors if they believe the risk premium is adequate

6. Higher risk premiums are associated with greater risk

B. Risk Aversion and Utility Values

1. History shows that investors are risk averse and require a risk premium to compensate for risk taken

2. Risk averse investors penalize expected returns for the risk involved

3. Choose among risky portfolios by assigning a utility score $U = E(r) - \frac{1}{2}A\sigma^2$, ($E(r)$ written as a decimal)

    a. Risk averse investors – $A > 0$
    b. Risk neutral investors – $A = 0$ (no penalty for risk)
    c. Risk lover – $A < 0$

4. Utility score can be interpreted as a certainty equivalent rate of return

    a. Investment is desirable only if certainty equivalent rate of return exceeds the risk-free rate

    b. Certainty equivalent rate – rate that a risk-free investment would need to offer to provide the same utility as the risky portfolio

5. Mean-Variance Criterion: Portfolio A dominates *B* if $E(r_A) \geq E(r_B)$ and $\sigma_A \leq \sigma_B$ and at least one inequality is strict

6. Indifference curve

    a. Curve connects points in the mean-standard deviation plane with the same utility values
    b. Investor has no preference amongst portfolios on the same indifference curve

C. Estimating Risk Aversion

1. Questionnaires

2. History of portfolio composition changes over time

3. Tracking of groups of individuals to determine averages

III. <u>Capital Allocation Across Risky and Risk-Free Portfolios</u>

A. Broad asset allocation between investment classes is most important part of portfolio construction

1. Treasury bills – low risk

2. Long-term bonds – moderate risk

3. Stocks – higher risk

B. First task is to determine allocation between a generic risky portfolio and the risk-free asset

1. The composition of the risky portfolio doesn't change with the amount invested in the risky portfolio, *y*

    a. Treat the risky portfolio as a single fund holding both bonds and equities in fixed proportion

IV. The Risk-Free Asset

A. Treasury bills are viewed as the most risk-free asset

1. Not truly risk-free in real terms, but government issued debt has default free guarantees that makes it the closest thing

2. The short-term nature makes it insensitive to interest rate fluctuations (can simply hold to maturity to lock in return)

B. Money market funds are also close to risk-free due to little default or credit risk and short maturities

1. Often comprised of the following:

a. Treasury bills
b. Other Treasury and U.S. agency securities
c. Repurchase agreements

V. Portfolios of One Risky Asset and a Risk-Free Asset

A. Expectations can be used to determine the rate of return of the complete portfolio, C

r_f = risk-free rate

r_P = rate on portfolio P

r_C = rate of complete portfolio

$E(r_P)$ = Expected return of portfolio P

$E(r_C)$ = Expected return of complete portfolio

σ_P = standard deviation of portfolio P

σ_C = standard deviation of complete portfolio

$$r_C = yr_P + (1-y)r_f$$

$$E(r_C) = r_f + y\left[E(r_P) - r_f\right] \qquad \sigma_C = y\sigma_P$$

B. The equations can be rearranged to define the return standard deviation trade-off:

$$E(r_C) = r_f + \sigma_C\left(\frac{\left[E(r_P) - r_f\right]}{\sigma_P}\right) = r_f + \sigma_C S$$

1. This is the equation of the line (**Capital Allocation Line**) passing through the risk-free portfolio and the risky portfolio in the return-variability plane

2. The slope of the line (**reward-to-volatility ratio a.k.a. Sharpe Ratio**) is given by the portion in brackets and provides the increased return for each additional unit of risk

C. If the investor can borrow at the risk-free rate it is possible to have values beyond the risky portfolio P along the CAL (same slope)

1. In reality only governments can borrow at the risk-free rate and other investors must borrow at a higher rate

a. In this case the CAL kinks at P, where the slope changes to: $\frac{\left[E(r_P)-r_f^B\right]}{\sigma_P}$, where

r_f^B = borrowing rate

b. Borrowing is typically done on margin with a broker

i. Margin purchases cannot exceed 50% of the purchase value
ii. Purchased securities must be maintained in a margin account and if the value declines below a maintenance margin, a deposit is required through a margin call

VI. Risk Tolerance and Asset Allocation

A. The formulas for utility (U) and expected return $\left[E(r_C)\right]$ and variance (σ_C) of the complete portfolio can be combined to determine the optimal allocation y^*

$$U = E(r) - \frac{1}{2} \text{ x } A \text{ x } \sigma^2$$

1. Utility is maximized by setting the first derivative with respect to y equal to 0 and solving:

$$y^* = \frac{\left[E(r_P)-r_f\right]}{A\sigma_P^2}$$

2. Optimal solution is directly proportion to the risky asset risk premium and inversely proportional to risk aversion

B. Utility curve analysis can be used to validate the optimal portfolio

1. A utility curve is a curve in the return-variability plane that describes all possible return-variability combinations having the same utility value

2. The indifference curve shows the expected return $(E(r))$ compared to the risk (σ^2)

3. Investors will always a prefer a higher indifference curve since for any given variability a higher curve corresponds to increased utility

4. The highest indifference curve that is tangent to the CAL will be tangent at y^*

C. Analysis thus far has considered returns to be normally distributed, which is frequently not the case

1. Probabilities of moderate outcomes can be assessed from experience

2. Probabilities of extreme negative outcomes are virtually impossible to estimate because they are very rare

VII. Passive Strategies: The Capital Market Line

A. Portfolio decisions that avoid any security analysis are known as passive strategies

1. Investing in value-weighted indices is the most common passive strategy e.g. S&P 500

2. This may be the reasonable strategy for many investors

   a. Cost to pursue an active strategy is not zero

   b. There is a free-rider benefit from following active investors who keep assets fairly priced by their buying and selling activities

3. Passive index funds may not be inferior to that of an average active investor

4. Passive investors allocate their investments according to their degree of risk aversion

B. The CAL that uses treasury bills as the risk-free asset and a broad market index (or proxy) for the risky asset is known as the **capital market line**

VIII. Appendix A: Risk Aversion, Expected Utility, and the St. Petersburg Paradox

A. The appendix tries to show that investors are risk averse

B. The St. Petersburg Paradox considers a coin toss game that has infinite expected returns, but most would only pay a small entry fee to partake

C. What needs to be considered is the utility gained or lost from partaking in such a game

1. The utility of an extra dollar varies inversely with wealth, i.e., the more wealth, the less utility of another dollar.

2. Rather than looking at the expected return in a game that might be 'fair,' consider the expected utility

3. If the expected utility of the outcome is less than the utility of the wager, a risk averse investor will reject it

4. The certainty equivalent value of the wager can be computed by determining the wealth that corresponds to the expected utility of the wager

D. Can use a log utility function to describe an investor's preferences.

IX. Appendix B: Utility Functions and Equilibrium Prices of Insurance Contracts
(combined with Appendix A in 12th edition)

A. A dollar in bad times (when wealth is low) is more valuable than a dollar in good times (when wealth is high)

B. The equilibrium value of a dollar in the low economy would be higher than the value of a dollar when the economy performs better than expected

C. Riskier bonds are sold at lower prices than safer ones with otherwise similar characteristics

D. Riskier stocks historically have provided higher rates of return over long periods of time

PAST CAS EXAMINATION QUESTIONS

A. Portfolios of One Risky Asset and a Risk–Free Asset

A1. Assume the expected market return is 15%, the Treasury bill rate is 6%, and that you can borrow at the risk-free rate. Further assume that you invest all your own money in the market portfolio, as well as half that amount of borrowed money. The expected return on your investment is:

A. ≤ 14% B. ≥ 14.1% but ≤ 16.0% C. ≥ 16.1% but ≤ 18.0% D. ≥ 18.1% but ≤ 20.0% E. ≥ 20.0% (90–5B–65–1)

A2. Assume the expected market return is 15% and the Treasury bill rate is 6%. Further assume that you invest 75% of your own money in the market portfolio, and lend the remaining 25% at a risk-free rate of interest. The expected risk premium on your investment is:

A. < 7.0% B. ≥ 7.0% but < 8.0% C. ≥ 8.0% but < 9.0% D. ≥ 9.0% but < 10.0% E. ≥ 10.0% (91–5B–54–1)

A3. You have $100 to invest and can borrow or lend money at the risk-free rate of 4%. The market portfolio offers an expected return of 12% with a standard deviation of 20%.

a. Combining borrowing, lending, and/or investing in the market portfolio, how can you construct a portfolio to achieve an expected return on your investments of 18%? Show all work.
b. What is the standard deviation of your portfolio? Show all work. (96F–5B–30–1/.5)

A4. Given the following regarding a risky portfolio (P) and the risk-free asset, calculate the slope of the capital allocation line for $\sigma < \sigma_P$.

$E(r_P) = 12\%$ $\sigma_P = 10\%$ $r_f = 5\%$

A. .05 B.07 C.10 D.50 E.70 (97–220–38)

A5. Company X invests all of its money in common stock portfolio A with an expected return and expected variance equal to that of the general market. Portfolio A is perfectly positively correlated with the market. The risk-free interest rate is 5% and the expected market risk premium is 8%. The standard deviation of returns of portfolio A is 20%. Calculate the expected return and standard deviation of company X's investments if the following investment strategies are implemented. Show all work.

a. Invest half of the money in portfolio A and half of the money in risk-free securities.
b. Borrow an amount equal to half of the company's current wealth at the risk-free rate and invest everything into portfolio A. (98F–5B–27–1/1)

A6. According to Bodie et al., each investor is content to put money into two benchmark investments.

a. Describe these two investments. Use a diagram to depict the investment strategy.
b. Explain how an investor can vary his/her expected rate of return and commensurate risk. (99S–5B–21–1/1)

A7. Given the following with respect to an optimal risky portfolio, calculate the slope of the capital allocation line (CAL) for this portfolio.

Expected return	7%	Risk premium	3%
Variance	100		

A. .03 B.04 C.30 D.40 E. 3.00 (03–6–1)

A1. r_C = (Percentage Invested in Market)$E(r_m)$ − (Percentage Borrowed)r_f
r_C = (1.5)(15%) − (.5)(6%) = 19.5%, pp. 172–73.

Answer: D

A2. Expected Investment Risk Premium = (Percentage Invested in Market)$(r_M - r_f)$
EIRP = (.75)(15% − 6%) = 6.75%, p. 171.

Answer: A

A3. a. r_C = (Percentage Invested in Market)$E(r_M)$ − (Percentage Borrowed)r_f

$.18 = (1 + x)(.12) - x(.04) \qquad x = .75$

Amount Borrowed = (.75)(Original Investment) = (.75)(100) = 75, pp. 172–73.

b. σ = (Percentage Invested in Market)(σ_M) = (1.75)(20%) = 35%, p. 171.

A4. $\text{Slope} = \dfrac{E(r_P) - r_f}{\sigma_P} = \dfrac{.12 - .05}{.10} = .7$, p. 172.

Answer: E

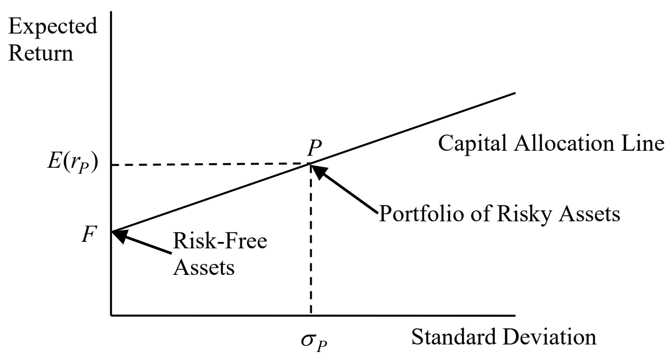
A5. a. $r_A = E(r_M) = r_f + \text{Risk Premium} = .05 + .08 = .13$

$E(r_C) = [E(r_A) + r_f]/2 = (.13 + .05)/2 = .09 \qquad \sigma_P = (.5)(\sigma_A) = (.5)(.20) = .10$, p. 171.

b. $E(r_C)$ = (Percentage Invested in A)$E(r_A)$ − (Percentage Borrowed)(r_f)
$E(r_C)$ = (1.5)(13%) − (.5)(5%) = 17%

σ_C = (Percentage Invested in A)(σ_A) = (1.5)(20%) = 30%, pp. 171–73.

A6. a. 1) Risk-free assets, i.e., Treasury bills
2) Portfolio of risky assets, i.e., the market portfolio



b. By changing the proportions of risk-free assets and the portfolio of risky assets in his portfolio, an investor can vary his expected return and associated risk. For example, if the proportion invested in risk-free assets is increased, his complete portfolio moves down the capital allocation line, reducing his expected return and lowering risk, p. 172.

A7. $\text{Slope} = \dfrac{E(r_P) - r_f}{\sigma_P} = \dfrac{3}{\sqrt{100}} = .3$, p. 172.

Answer: C

A8. Consider the following information about a risky portfolio and a risk-free asset.

i) The risk premium of the risky portfolio is 15%.
ii) The reward-to-variability ratio of the risky portfolio is .75.
iii) The expected return on the risk-free asset is 3.0%.

Assume you can invest in some combination of the risky portfolio and the risk-free asset. Determine the equation for the capital allocation line (CAL) under these assumptions and graph the CAL. Label all items properly. Show all work. (06–8–4–2)

A9. You are given the following information:

i) A risky portfolio has an expected return of 16% and a standard deviation of 25%.
ii) The T-bill rate is 6%.

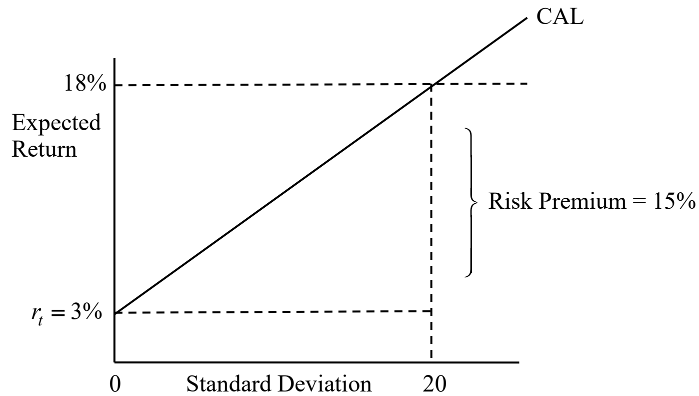
a. Suppose you invest 60% of your funds in the risky portfolio and 40% in a T-bill money market fund. Calculate the expected value and the standard deviation of the rate of return of the portfolio.
b. Determine the equation of the capital allocation line (CAL) of the risky portfolio and graph the CAL. Plot the position of the overall portfolio on the CAL graph. Label all items properly.

Show all work. (07–8–1–.5/1)

A10. i) The return of a risk-free asset is 5%.
ii) An investment company offers a risky asset, with a Sharpe ratio of .2.
iii) An investor wants to hold a portfolio consisting of the risky asset and the risk-free asset.

a. Calculate the expected return of the portfolio if the investor wants the standard deviation of the portfolio to be 15%.
b. Graph the capital allocation line (CAL) associated with this portfolio. Plot the position of the overall portfolio on the CAL graph. Clearly label the axes, the CAL, the risk-free asset, and the overall portfolio. (10–8–2–.5/.75)

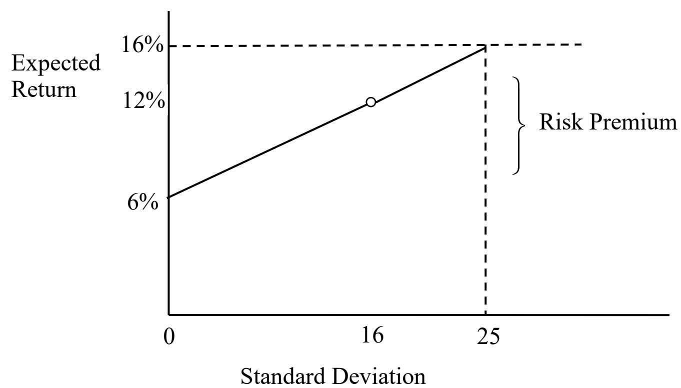
A8. $E(r) = r_f + (\text{Reward-Variability Ratio})\sigma = .03 + .75\sigma$



p. 172

A9. a. $E(r_C) = r_f + y[E(r_P) - r_f] = .06 + (.60)(.16 - .06) = .12$
$\sigma_C = y\sigma_P = (.60)(.25) = .15$, p. 171.

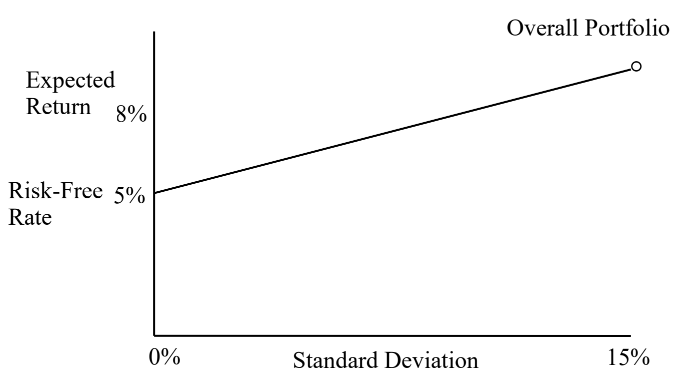
b. $E(r_C) = r_f + [E(r_P) - r_f][\sigma_C/\sigma_P] = .06 + (.16 - .06)(\sigma_C/.25) = .06 + .4\sigma_C$



p. 172

A10. a. $E(r_P) = S\sigma_P + r_f = (.2)(.15) + .05 = .08.$
p. 172

b.



p. 172

A11. Given the following about a risky portfolio, P:

- Expected return of P is 16%.
- Standard deviation of expected return of P is 24%.
- Borrowing rate is 10%.
- Lending rate is 6%.

Graph the capital allocation line (CAL). Plot the position of P on the CAL graph. Clearly label the axes, the y-intercept of the CAL, and the borrowing and lending ranges.

(14-9-1-1.25)

A12. The values in the table below were empirically estimated.

Security	Expected Return	Beta
Stock A	6%	0.5
Stock 8	10%	1.5

- The expected market return is 8%.
- The risk-free rate is 2%.

a. On a single graph, draw and label:

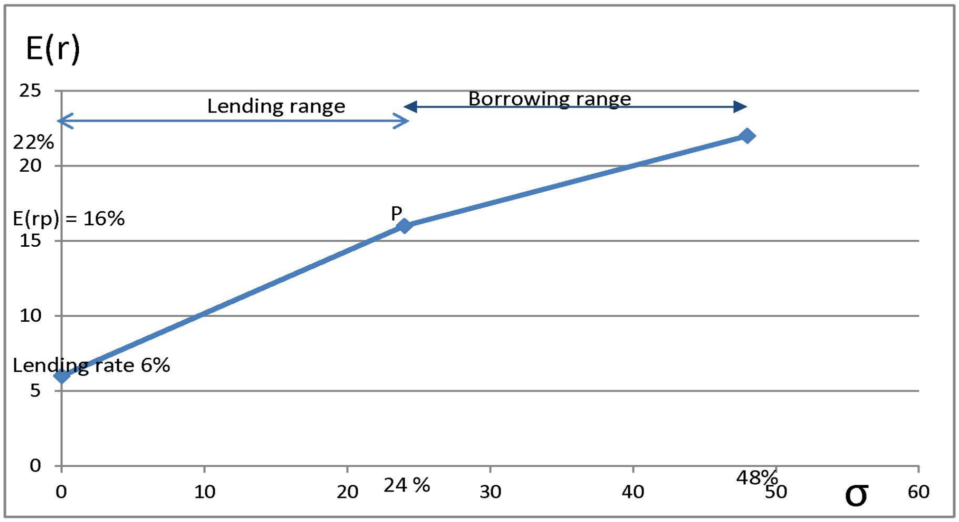
i. The Security Market Line (SML) implied by the expected market return.
ii. The empirically estimated SML.
iii. The risk-return point for both securities listed above.

b. Explain whether Stock A or Stock 8 is a better buy according to the Capital Asset Pricing Model (CAPM).

(14-9-3-1.5/0.5)

A11.

expected model solution overall



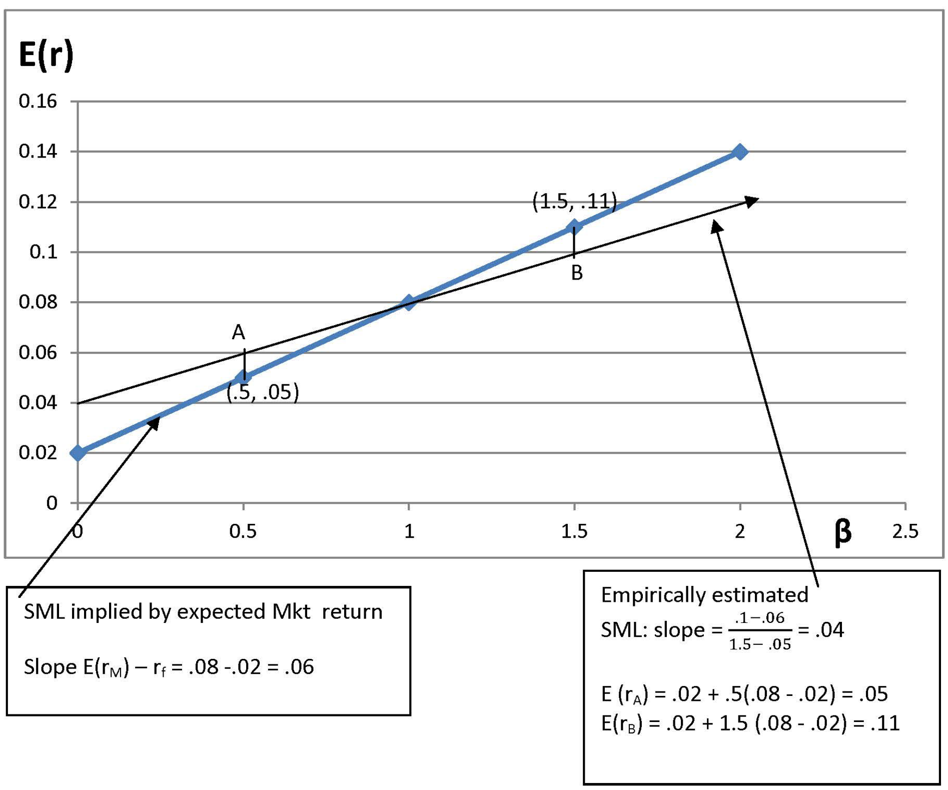
A12. a.

SML $E(r_i) = r_f + \beta(E(r_m) - r_f)$

$E(r_m) = .08$

$r_f = .02$

$B = 1$ for r_m



b. Stock A has a positive α, $\alpha A = 1\% = 6\% - [2\% + 0.5\,(8\% - 2\%)]$
Stock B has a negative α, $\alpha B = -1\% = 10\% - [2\% + 1.5(8\% - 2\%)]$
Therefore, Stock A is a better buy based on CAPM.

A13. An investor has a risk aversion factor of A = 2. The investor has $100,000 to invest in a combination of an S&P 500 index fund and risk-free assets. Given the following information:

- The investor's utility is given by $U = E(r_0) - 0.5A\sigma_0^2$.
- The investor's risk-free lending rate is 4%.
- The investor's risk-free borrowing rate is 6%.
- The S&P 500 index offers an expected return of 11.5% with a standard deviation of 20%.

a. The investor would like to achieve the greatest possible return, subject to a maximum standard deviation of 25%. Calculate the amount the investor should allocate to risk-free assets.

b. Calculate the expected return and standard deviation of the portfolio in part a. above.

(16-9-1-0.5/0.75)

A14. a. Describe what a passive investment strategy is.

b. Briefly describe two advantages of selecting a passive investment strategy over an active strategy.

(17-9-2-0.5/0.5)

A13. a. Since the expected return of the S&P 500 index exceeds the risk-free borrowing rate, the investor should allocate as much funds as possible to achieve the greatest expected return.

$$0.25 = y^*.2 \rightarrow y^* = 1.25$$
$$(1 - 1.25)(\$100{,}000) = -\$25{,}000$$

The investor should borrow $25,000 at the risk-free rate

b. $E(r_c) = y^* E(r_p) +)1 - y^* E(r_f)$ = (1.25)(0.115)+(-0.25)(0.06)=12.88%

$\sigma_c = y^* \sigma_p =$ (1.25)(0.2))=0.25

A14. a. Passive investment strategy relies on investing in a well-diversified portfolio often mimicking a market index without actively researching/security analysis to find mispriced assets.

b. 1) **Transaction costs** – Passive strategy is cheaper than active strategy – don't need to spend money researching stocks and constantly buying and selling.

2) **Free-rider benefit** – Passive strategy benefits from work of active portfolio managers who do security analysis and make stock prices more accurate by identifying mispricing and trading based on them.

B. Risk Aversion and Risk Tolerance

B1. The utility of a risk-free portfolio is determined by the expected rate of return of the portfolio. (93–220–25)

B2. Risk-averse investors have steeper indifference curves than risk-neutral investors. (94–220–28)

B3. Given the following, calculate the optimal percentage the investor should invest in the market.

Expected market return	12%	Standard deviation of the market return	10%
Risk-free rate	4%		

The utility function of the investor $U_M = E(R_M) - \sigma(R_M)^2/5$.

A. 10% B. 20% C. 30% D. 40% E. 50% (02–6–29)

B4. You are given the following with respect to two portfolios:

	Portfolio A	Portfolio B
Expected return	10%	8%
Variance	2%	3%

An investor has all his funds in portfolio A. His expected utility is the same as for a certain return of 6%. Calculate the equivalent return for portfolio B for this investor.

A. −.50% B. 2.00% C. 3.00% D. 3.67% E. 6.50% (03–6–9)

B5. Given the following, determine whether or not the investor would purchase stock X. Show all work.

	Profitability	One-Year Return
Stock X	.60	10%
	.20	5%
	.20	−10%
Stock Y	.75	20%
	.25	−20%

i) The risk-free rate is 4%.
ii) The investor has a one-year horizon.
iii) The investor is indifferent between investing in stock Y and earning the risk-free rate. (03–6–W2–5)

B6. Answer the questions below based on the following information about a risky portfolio that you manage, and a risk-free asset:

$E(r_P) = 11\%$ $\sigma_P = 15\%$ $r_f = 5\%$

a. Client A wants to invest a proportion of her total investment budget in your risky fund to provide an expected rate of return on her overall or complete portfolio equal to 8%. What will be the standard deviation of the rate of return on her portfolio?

b. Client B wants the highest return possible subject to the constraint that you limit his standard deviation to be no more than 12%. Which client is more risk averse? Explain why.

Show all work. (03–8–4–.5ea.)

B1. T, p. 164.

B2. T, pp. 163–64 – For a risk-averse investor, the expected return increases with the standard deviation. For a risk-neutral investor, the expected return is constant.

B3. $.5A = .2 \qquad A = .4 \qquad y^* = \dfrac{E(r_P) - r_f}{A\sigma_P^2} = \dfrac{12 - 4}{(.4)(10)^2} = 20\%$, p. 163.

Answer: B

B4. 1) Calculate the risk aversion coefficient:

$$6\% = U_A = E(r_A) - A\sigma_A^2 = 10\% - (A)(2\%) \qquad A = 2$$

2) Calculate the utility for portfolio B:

$U_B = 8\% - (2)(3\%) = 2\%$, p. 163.

Answer: B

B5. 1) Calculate the mean and variance for stock Y:

$$E(r_Y) = (20\%)(.75) + (-20\%)(.25) = 10\%$$
$$\sigma_Y^2 = (20\% - 10\%)^2(.75) + (-20\% - 10\%)^2(.25) = 3\%$$

2) Calculate the risk aversion coefficient:

$$4\% = U_Y = E(r_Y) - A\sigma_Y^2 = 10\% - (A)(3\%) \qquad A = 2$$

3) Calculate the mean and variance for stock X:

$$E(r_X) = (10\%)(.60) + (5\%)(.20) + (-10\%)(.20) = 5\%$$
$$\sigma_X^2 = (10\% - 5\%)^2(.60) + (5\% - 5\%)^2(.20) + (-10\% - 5\%)^2(.20) = .6\%$$

4) Calculate the utility for portfolio X:

$$U_X = 5\% - (2)(.6\%) = 3.8\%$$

Since the utility for portfolio X is less than that for a risk-free asset (4%), the investor would not purchase it, p. 163.

B6. a. $y = \dfrac{8\% - r_f}{E(r_p) - r_f} = \dfrac{8\% - 5\%}{11\% - 5\%} = .5$

Since the rest of her portfolio is invested in the risk-free asset, the formula for the portfolio variance reduces to the following:

$$\sigma_A = y\,\sigma_P = (.5)(15\%) = 7.5\%$$

b. For B to earn a return of 12%, he has to invest more than 50% of his portfolio in the risky asset and thus A, who has a lower percentage in the risky fund, is more risk averse, pp. 163, 171.

B7. The following question consists of an assertion and a reason:

Assertion – Given the choice, investors prefer a portfolio on a higher indifference curve.
Reason – Higher indifference curves correspond to higher levels of utility.

A. Both the assertion and the reason are true statements, and the reason is a correct explanation of the assertion.
B. Both the assertion and the reason are true statements, but the reason is not a correct explanation of the assertion.
C. The assertion is a true statement, but the reason is a false statement.
D. The assertion is a false statement, but the reason is a true statement.
E. Both the assertion and the reason are false statements. (04–6–21)

B8. You are given the following information:

Expected return of the risky asset (E(r))	.13	Risk-free rate	.06
Variance of the risky asset (σ^2)	.01	Coefficient of risk aversion (A)	5

Utility function $U = E(r) - .01A\sigma^2$

a. Calculate the expected return and standard deviation of a portfolio that is invested 40.0% in the risky asset and 60.0% in a risk-free asset.
b. Determine the optimal amount to invest in each asset to maximize the utility.

Show all work. (04–8–3–1/2)

B9. You are given the following information.

Investment	Expected Return	Standard Deviation
1	12%	20%
2	15%	30%
3	20%	40%

Your utility formula is represented by $U = E(r) - .003A\sigma^2$.

a. Briefly explain which of the three investments a risk-neutral investor would select.
b. Identify which investment an investor with the utility function shown above and A = 2 would select.
c. Calculate the certainty equivalent of the investment selected in b. (05–8–4–.5/.5/.75)

B10. Given the following information regarding a risk-free asset and a portfolio of risky assets:

i) The risk-free rate is 3%.
ii) The expected return on the risky portfolio is 11%.
iii) The standard deviation of the risky portfolio's return is 25%.
iv) An investor has utility function $U = E(r) - A\sigma^2/3$ with risk aversion parameter A = 2.

This investor has a $50,000 budget for investing. Calculate how much the investor should invest in the portfolio of risky assets to maximize the investor's utility. Show all work. (09–8–1–1.75)

B7. Both the assertion and the reason are true, and the reason is a correct explanation of the assertion, p. 176.

Answer: A

B8. a. $E(r_C) = r_f + y[E(r_P) - r_f] = .06 + (.40)(.13 - .06) = .088$

$\sigma_C = y\sigma_P = (.40)(.1) = .04$, p. 171.

b. Since the coefficient before A in the utility formula is .01, the appropriate formula is:

$$y^* = \frac{E(r_P) - r_f}{.02A\sigma_P^2} = \frac{13 - 6}{(.02)(5)(10)^2} = 70\%$$

Invest 70% in the risky asset and 30% in the risk-free asset, p. 175.

B9. a. A risk-neutral investor would select the investment with the highest expected return., i.e., investment 3, p. 164.

b. $U = E(r) - .006\sigma^2$

$U_1 = 12 - (.006)(20)^2 = 9.6\%$
$U_2 = 15 - (.006)(30)^2 = 9.6\%$
$U_3 = 20 - (.006)(40)^2 = 10.4\%$

The investor would select investment 3, p. 163.

c. Its certainty equivalent is its utility value, i.e., 10.4%, p. 163.

B10. $$y^* = \frac{E(r_P) - r_f}{.02A\sigma_P^2} = \frac{11 - 3}{(.02)(2/3)(25)^2} = 96\%$$

Invest (.96)(50,000) = 48,000 in the risky asset and 50,000–48,000 = 2,000 in the risk-free asset, p. 175.

B11. Given the following information regarding a risk-free asset and a risky asset

i) The return of the risk-free asset is 5%
ii) An investment company offers a risky asset, with an expected return of 12% and a standard deviation of 15%.
iii) An investor has the utility function $U = E(r) - 3A\sigma_C^2$ with risk aversion parameter A = 1.5.
iv) The investor wants to create a portfolio using the risk-free asset and the risky asset.

Calculate the proportion of the portfolio comprised of the risky asset that maximizes the investor's utility. (10–8–3–1.75)

B12. A portfolio is being constructed for an investor using the following assets and assumptions:

	Assets		
	D	E	F
Expected Return	0.12	0.14	0.04
Standard Deviation	0.15	0.10	0.00

- The utility function is $U = E(r_C) - 0.5A\sigma_C^2$
  - The subscript *c* refers to the optimal portfolio
- The coefficient of risk aversion *A*, is 7
- The weight given to asset *D* in the optimal risky portfolio is 0.16
- The reward-to-volatility ratio is 1.03
- The investor is allowed to borrow at the risk-free rate

a) Calculate the share of the optimal complete portfolio invested in the risk-free asset that would maximize the investor's utility.
b) Describe the result in part a) above in terms of the optimal risky portfolio. (11-9-1-1.75/0.5)

B13. Given the following information about investment options:

1.1. $U = E(r_C) - 0.5A\sigma_C^2$
1.2. The risk-free rate of return is 3%
1.3. The risk premium on the risky portfolio is 5%.
1.4. The reward-to-volatility ratio of the risky portfolio is 0.25.
1.5. The risk aversion parameter is 2.

a) Calculate the certainty equivalent rate of the risky portfolio.
b) Given that 40% of the available assets are invested in the risky portfolio and the rest in the risk-free asset, calculate the expected return and standard deviation of the complete portfolio.

Calculate the percentage of available assets to invest in the risky portfolio to maximize utility.

(13-9-1-0.75/0.5/0.25

B11. Since the coefficient before A in the utility formula is 3, the appropriate formula is:

$$y^* = \frac{E(r_P) - r_f}{.06As_P^2} = \frac{12 - 5}{(.06)(1.5)(15)^2} = .34568$$

Invest 35% in the risky asset and 65% in the risk-free asset, p. 175.

B12. a) $r_f = 0.04$ (the risk-free return)

$w_D = 0.16, \quad w_E = 1 - w_D = 0.84$

$E(r_P) = w_D E(r_D) + w_E E(r_E) = 0.16(0.12) + 0.84(0.14) = 0.1368$

Use the reward-to-volatility ratio to calculate σ_P (page 172):

$$\frac{E(r_P) - r_f}{\sigma_P} = \frac{0.1368 - 0.04}{\sigma_P} = 1.03, \quad \sigma_P = 0.0940$$

Optimal weight in the risky portfolio is given by y^* (page 175):

$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.1368 - 0.04}{7(0.0940)^2} = 1.565$$

Invest weight of $1 - y^* = -.565$ in the risk-free portfolio

b) For optimal allocation the investor needs to borrow at the risk-free rate and invest the money in the risky-portfolio

B13. a) The certainty equivalent rate of return is the utility score of the risky portfolio pp. 163:

$$E[r_p] = 0.05 + 0.03 = 0.08$$

$$S = 0.25 = \frac{E[r_p] - r_f}{\sigma_p} \rightarrow \sigma_p = 0.2$$

$$U_p = 0.08 - .5(2)(0.2)^2 = 0.04$$

b) $E[r_c] = 0.6(0.03) + 0.4(0.08) = 0.05$

$$\sigma_c^2 = \sqrt{0.4^2(0.2)^2} = 0.08$$

c) The optimal amount invested in the risky portfolio is given by $y^* = \frac{E(r_P) - r_f}{A\sigma_P^2}$

$$y^* = \frac{0.08 - 0.03}{(2)(0.2)^2} = 0.625 = 62.5\%$$ invested in the risky portfolio.

B14. Contrast active and passive investment strategies regarding:

i. constructing a portfolio.
ii. implementation and total costs.
iii. expected return.

(14-9-2-1.5)

B14. i) An active strategy requires analysis of stock prices in an effort to seek out mispriced stocks and add underpriced stocks to the portfolio while removing or adding short positions on overpriced stocks. Analysis may be technical or fundamental. On the other hand, a passive strategy would invest in the overall market or a market index as the risky portfolio and adjust only the amounts invested in the risky portfolio and the risk-free asset based on the investors risk appetite.

ii) An active strategy will cost much more than a passive strategy. There are costs of doing the analysis that apply both at implementation and as long as the portfolio is held, and there are transaction costs each time the portfolio is changed. The passive strategy, however, is relatively cheap as there is no need for analysis and minimal transaction cost.

iii) The two strategies are likely to provide similar expected return. On average, it is not possible to outperform the market as the market is all investors. Any excess returns earned through an active strategy are likely to be offset by the associated costs.

Bodie Kane Marcus

Investments, Twelfth Edition: Chapter 7
Efficient Diversification

OUTLINE

I. Notation

A. $E(r)$ Expected return

B. σ^2 Variance of returns

C. A Index of investor's risk aversion

D. P Portfolio of risky assets

E. r_X Risky portfolio total rate of return of portfolio X

F. $E(r_x)$ Expected rate of return of portfolio X

G. σ_X Standard deviation of portfolio X

H. r_f Risk-free return

I. w_E Proportion of risky portfolio in equities

J. w_D Proportion of risky portfolio in bonds

K. S_p Sharpe ratio

L. R_X Risky portfolio excess rate of return on portfolio X

M. $\overline{Cov}$ Average covariance between securities in a portfolio

N. $\overline{\sigma^2}$ Average variance of securities in a portfolio

O. y Proportion of investment in risky portfolio

P. CAL Capital allocation line

Q. ES Expected shortfall

II. Diversification and Portfolio Risk

A. Two broad categories of risk to a portfolio:

1. Market Risk (aka systematic risk, nondiversifiable risk)

   a. Conditions in general economy
      i. Business cycle
      ii. Inflation
      iii. Interest rates
      iv. Exchange rates

2. Firm-specific Risk (aka unique risk, nonsystematic risk, diversifiable risk)

B. Diversification by purchasing differing securities reduces total portfolio risk by reducing firm-specific risk

1. Naïve diversification holds equal weights of various securities

2. Market risk can't be diversified away

3. When risk sources are independent, risk can be reduced to a very low level through diversification - This is known as the **insurance principle**

III. Portfolios of Two Risky Assets

A. Portfolios of two risky assets can be combined with the following formulae

$$E(r_P) = w_D E(r_D) + w_E E(r_E)$$
$$\sigma_P^2 = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E Cov(r_D, r_E) = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \sigma_D \sigma_E \rho_{DE}$$

B. Formulas for the portfolios with more than two variables can be determined through construction of a bordered covariance matrix

1. First row and column of the matrix has each of the weights listed with corresponding covariance of returns

2. Each covariance in the table is multiplied by the weights in its row and column borders and summed

C. Correlation coefficient will determine the degree of diversification benefit

1. If two securities are perfectly correlated $(\rho = 1)$ there will be no diversification benefit

2. Anything less than perfect correlation will lead to a standard deviation that is less than that of the weighted sum of standard deviations (demonstrates diversification benefit)

3. If securities are perfectly negatively correlated $(\rho = -1)$ then it is possible to achieve a perfectly hedged position with weights given by:

$$w_D = \frac{\sigma_E}{\sigma_D + \sigma_E} = 1 - w_E$$

4. Can determine the **minimum-variance portfolio** for any two securities (regardless of correlation) with the following weights:

$$W_{Min}(D) = \frac{\sigma_E^2 - Cov(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2Cov(r_D, r_E)} = 1 - w_{Min}(E)$$

5. A minimum-variance portfolio has a standard deviation smaller than that of either of the individual component assets

D. Can construct a **portfolio opportunity set** by plotting all portfolios in the risk-variability plane for a given correlation coefficient

1. Use various weights and calculate the return and standard deviation then plot

E. For a given correlation coefficient the optimal portfolio (where the total portfolio is constructed of two risky assets) can be taken from the opportunity set by incorporating equations for utility (chapter 6 of BKM) to give:

$$w_D = \frac{E(r_D) - E(r_E) + A(\sigma_E^2 - \sigma_D \sigma_E \rho_{DE})}{A(\sigma_D^2 + \sigma_E^2 - 2\sigma_D \sigma_E \rho_{DE})} = 1 - w_E$$

IV. Asset Allocation with Stocks, Bonds and Bills

A. Purpose of this section is to tie together the allocation of an investment between the risk-free asset and the risky portfolio and the construction of that risky portfolio

B. When there is the option of investing in a risk-free asset the optimal weights for the risky portfolio will occur where the CAL is tangent to the risky portfolio opportunity set

1. This maximizes the Sharpe ratio

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

2. The optimal risky portfolio weights found through maximizing the equation for the reward-to-volatility ratio are given by:

$$w_D = \frac{E(R_D)\sigma_E^2 - E(R_E)Cov(R_D, R_E)}{E(R_D)\sigma_E^2 + E(R_E)\sigma_D^2 - [E(R_D) + E(R_E)]Cov(R_D, R_E)} = 1 - w_E$$

3. The weights are then used to determine the return and variability of the optimal risky portfolio using the equations above

C. Once the optimal risky portfolio is constructed, the equation for utility from chapter 6 can be used to incorporate investor risk tolerance and allocate between the risk-free asset and the optimal risky portfolio, recall the weight in the risky portfolio: $y = \dfrac{E(r_P) - r_f}{A\sigma_P^2}$

1. The amount invested in the risk-free asset is $(1 - y)$
2. The amount invested in Bonds is $y(w_D)$
3. The amount invested in Equities is $y(w_E)$

V. The Markowitz Portfolio Selection Model

A. Portfolio construction can be generalized to the case where there are many risky assets and a risk-free asset

B. Security selection

1. First step is to determine the minimum-variance frontier

a. Need list of inputs (expected return, standard deviation and covariance with other assets) for all securities (text assumes this knowledge is known)

   b. **Minimum-variance frontier** is the lowest variance that can be achieved for a given expected return using all available securities and allowing short selling (use bordered covariance matrix for each possible portfolio or the following formula): $\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$

   c. The points above the global minimum-variance portfolio form the **efficient frontier of risky assets** (for any risk level only interested in portfolio with highest possible return)

2. Restrictions are sometimes placed on the portfolios available, such as

   a. no short-selling

   b. minimum dividend yield

   c. ruling out industries or countries considered ethically or politically undesirable (**socially responsible (SRI) investing**)

   d. requiring **environmental, social, and governance-focused (ESG)** investing that focuses on long-term sustainable business practices that might be enhanced by environmentally and socially sound practices – some overlap with SRI investing

3. Any constraints result in a Sharpe ratio inferior to that of a less-constrained one

C. Capital Allocation and Separation Property

1. Need to find the CAL that lies tangent to the efficient frontier of risky assets to determine the optimal risky portfolio

   a. Maximize the slope of CAL (reward-to-volatility ratio)

   b. This will be the same for all investors – so a fund manager offers one portfolio to all investors

2. Separation property states that portfolio selection is comprised of two tasks:

   a. Determine the optimal risky portfolio (same for all investors)

   b. Determine allocation between risk-free asset and risky portfolio (differs based on risk aversion)

3. In reality there is more than one risky portfolio because:

   a. different managers will have differing results from analysis (return, variability, correlation, etc.)

   b. different investors will have different constraints so the efficient frontier is different

4. In theory one risky portfolio can serve a large number of people, which is the basis for the mutual fund industry

D. Power of Diversification

1. Formula for variance of a portfolio: $\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$

2. With naïve diversification strategy (equally weighed portfolio) the variance becomes:

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma^2} + \frac{n-1}{n}\overline{Cov}$$

3. Use this formula to show the power of diversification:

    a. As n gets large the variance of the portfolio approaches the average covariance of the securities

        i. If the securities are uncorrelated the variance approaches 0
        ii. In reality some correlation will exist among the securities and the portfolio risk will be positive
        iii. Higher degrees of correlation reduce the total amount of diversification possible and also the marginal benefit from each additional security included

4. The contribution to the portfolio risk of a particular security will depend on the covariance of that security's return with those of the other securities in the portfolio and not on the security's variance

E. Asset Allocation and Security Selection

1. Why differentiate between security selection and asset allocation?

    a. Demand for sophisticated analysis has increased with greater need to save

    b. Sophisticated analysis beyond the capacity of amateur investors

    c. Economies of scale and decentralization of portfolio management in what have become large investment firms

F. Optimal Portfolios and Nonnormal Returns

1. Diversification is more difficult to quantify in the case where returns do not follow a normal distribution

2. Value at Risk (VaR) and Expected Shortfall (ES) can be calculated using bootstrapping

VI. <u>Risk Pooling, Risk Sharing, and Time Diversification</u>

A. Risk sharing versus risk pooling

1. Risk pooling for investing means adding uncorrelated risky projects to the investor's portfolio

2. Risk pooling for insurance means selling many uncorrelated insurance policies

3. Insurance example of risk pooling

    a. Payout on any particular policy is \$$x$ with the variation of x is σ^2

    b. Total payout is $\sum_{i=1}^{n} x_i$

    c. If risks are uncorrelated, $Var\left(\sum_{i=1}^{n} x_i\right) = n\sigma^2$

    d. Variance of the average payour is $Var\left(\frac{1}{n}\sum_{i-1}^{n} x_i\right) = \frac{1}{n^2} \times n\sigma^2 = \frac{\sigma^2}{n}$

    e. As the insurer writes more policies, n increases and the uncertainty of the average payout decreases

    f. However, undercertainty of the total payout increases

4. Same with gambling in a casino

5. Risk pooling is part of the insurance model; the other part is risk sharing

6. Risk sharing allows the investor to maintain the same size of investment while realizing the benefit of risk pooling

7. Insurance example of risk sharing

    a. Same as in 3, but the insurer sells *n* policies and has *n* investors

    b. The undercertainty of the average payout decreases as before

    c. As *n* increases, both the number of policies and number of investors increase, so each investor's investment is unchanged and the uncertainty of the average payout remains the same for each investor.

8. Diversifying a portfolio means dividing a fixed investment across many assets.

9. True diversification occurs in a portfolio of fixed size across many assets, not by adding more assets

B. Time Diversification

1. Investing longer term (say two years instead of one) is analogous to investing in two uncorrelated assets (assuming one year is not related to the next) or selling two uncorrelated insurance policies

    a. This is a form of risk pooling

    b. There is less risk in doing this then investing double in the first year and then investing risk-free in the second year

2. Total risk still increases since it is a risk pooling arrangement

3. To diversify need to share a portion of the portfolio across years

VII. Appendix A – Spreadsheet Model for Efficient Diversification

VIII. Appendix B – Review of Portfolio Statistics (in a Spreadsheet)