ACTEX Learning

Flashcards

for

ILA Life ALM & Modeling

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Section 1

Stochastic Models, Generalized Linear Models, Multi-State and Transition Matrix Models

Risk Models

- Risk models give managers insight into the source of risk in a portfolio and help them control their exposures and understand contributions of different portfolio components to total risk

- Risk models help managers in the decision-making process as to how to make changes to the portfolio

Handbook of Fixed Income Securities, ch. 49

Portfolio Volatility Approach

- The portfolio return is a function of individual instrument returns and the market weights of the securities in the portfolio

- The forecasted volatility of the portfolio σ_P^f can be computed as a function of these weights w and the covariance matrix Σ_S of the instrument returns in the portfolio

$$\sigma_P^f = \text{sqrt}\,(w^T \times \Sigma_S \times w) \quad \text{where } T \text{ denotes the matrix transpose}$$

- This covariance matrix can be decomposed into the individual instrument volatilities and correlations among returns

- Using this approach, the portfolio manager gains insight into the riskiness of the portfolio and how the portfolio can be diversified

Handbook of Fixed Income Securities, ch. 49

Multi-factor Models

- A major characteristic of multi factor models is their ability to provide a return to the portfolio using a small set of variables called factors which are designed to capture broad systematic market fluctuations and the nuances of individual portfolios
- Most factor models are linear in that the total return can be decomposed into the sum of the contributions of the factors
    - Systematic return is a component of total return due to movements in the common risk factors
    - Idiosyncratic return can be described as a residual component that cannot be explained by systematic factors systematic factors
- Correlations across securities of different issuers are driven by their exposures to the systematic risk factors and the correlation between those factors

Handbook of Fixed Income Securities, ch. 49

Equations for Portfolio Total Return and Volatility

- The systematic and idiosyncratic components of total return for a security *s* where the systematic return is a product of the instrument's loadings to the systematic risk factors *F* and the returns of these factors, and the idiosyncratic return is ε_s

$$R_s = L_s \times \mathrm{F} + \varepsilon_s$$

- Portfolio volatility can be estimated as

$$\sigma_P^f = \mathrm{sqrt}\ (L_p^T \times \Sigma_F \times L_p + w^T \times \Omega_S \times w)$$

where L_p is the loading to the portfolio to the risk factors
Σ_F is the covariance matrix of factor returns
Ω_S is the covariance matrix of the idiosyncratic security returns

Handbook of Fixed Income Securities, ch. 49

Advantages of Multi-factor Models

- The idiosyncratic return of securities is diversified away as the number of securities in the portfolio increases (the diversification benefit)

- A more robust estimation is created because the number of parameters is smaller in Σ_F is smaller than Σ_s

- Factors can be designed to be more stable than individual stock returns leading to models with better predictability

- They provide insight into the structure and properties of the portfolio and enable decision making

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Event Risk

- Even sophisticated models are subject to event risk caused by geopolitical or financial events which are followed by a period of

    - Large negative returns of risky assets
    - Large positive returns of assets considered safe and
    - Significantly higher volatility and very high correlations

- Event risk will not be captured in the projections and must be tested using a “what if” analysis under stress conditions

Handbook of Fixed Income Securities, ch. 49

Systematic Fixed Income Risk Factors

- Risks with influence across asset classes (yield curve)
- Sector specific to a particular asset class (prepayment)

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Global Risk Model

- Risk factors are other observable or estimated from regressing cross-sectional asset returns on instrument sensitivities

- The forecasted systematic risk is a function of the mismatch between the portfolio and the benchmark, and the exposure is to the risk factors such as spreads

- Net portfolio exposures are aggregated from security level analytics

- Systematic risk is also a function of volatility of risk factors and the correlations between risk factors

- Because the model uses security level returns and analytics to estimate the factors, it can recover the idiosyncratic return for each security

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Curve Risk Benchmarks

- Curve risk is the major source of risk across fixed income instruments; this matches curve profiles relative to a benchmark and is usually the main driver of portfolio risk

- Typically, the benchmark is the government or the swap curve

    - During calm periods the behavior of the swap curve tends to match the government curve

    - However, during a liquidity crisis they can diverge significantly

- For government-based products the government curve is used; for all other products the spreads between the swap and the government curve are used

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Credit Risk

- Bonds issued by corporations have credit risk, and holders of these securities demand additional yield above the risk-free rate to compensate for that risk

- The risk is usually measured by reference to a curve such as a swap curve; the total of credit spreads determines the credit risk exposure associated with the portfolio

- Characteristics of credit bonds have systematic sources of credit spread risk such as the industry, credit ratings and the country of the issuer

- The loading of a bond to a credit risk factor would commonly use spread duration multiplied by the bonds spread

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Prepayment Risk Model

- The model captures interest rates using key rate durations and their convexity

- Convexity is generally negative and has a detrimental effect on the market value of the instrument when interest rates move in either direction

    - Decreasing interest rates cause prepayments to increase thereby reducing price appreciation due to falling rates

    - Rising interest rates intensify the price depreciation the instrument suffers with the higher rates

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Implied Volatility Risk

Many fixed income securities have embedded options for interest rates or the discount curve used to price the security; if expected volatility increases, options generally become more expensive thereby affecting the prices of bonds with embedded options

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Liquidity Risk

- Fixed income securities traded in decentralized markets are illiquid, making it difficult to establish a fair price

- These bonds are exposed to liquidity risk because the illiquid bondholder may have to pay a higher price to liquidate a position, generally selling at a discount

- The amount of the liquidity discount is uncertain and changes during the business cycle

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Inflation Risk

- Inflation securities are based on expectation of future inflation

- This uncertainty adds to the volatility of a bond over the volatility of other risk sources

- Expected inflation is not an observable variable in the marketplace but can be extracted from prices of inflation linked government bonds and inflation swaps

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Tax Policy Risk

- Many securities are tax exempt which provide an additional benefit based on the allowable tax exemption that is incorporated in the price of the security

- Uncertainty around tax policy adds to the risk of these securities

- This cannot be observed in the marketplace and must be extracted from the prices of municipal securities

- The return on municipal securities in excess of interest rates is driven by tax policy expectations and the creditworthiness of the issuer which can be difficult to separate

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Idiosyncratic Risk

- Once all systematic factors and holdings are determined, the residual idiosyncratic return of a security can be computed as the component of its total return that cannot be explained by systematic factors

- Idiosyncratic return can be a significant component of total return but tends to decrease rapidly as the number of instruments increases

- The major inputs to idiosyncratic risk are the instrument characteristics and historical idiosyncratic returns of the instruments, such as instrument spread, spread duration, industry membership, and idiosyncratic volatility

- Idiosyncratic returns of different issuers are considered to be uncorrelated; however, different securities from the same issuer can have a level of co-movement

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Portfolio Rebalancing

- Most managers rebalance their portfolios at regular intervals to reflect changing views and market circumstances

- Over time, statistics drift from targeted levels due to aging of the holdings, changes in the market environment, or issuer specific events

- A risk model is very useful in rebalancing a portfolio

- A risk model can tell the manager how much risk reduction a transaction can achieve in order to evaluate the risk reduction relative to transaction cost

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Scenario Analysis

- Mechanics for a scenario return of the portfolio
  - The manager translates his views into risk factors
  - The covariance matrix is used to complete all risk factors
  - Net loadings of all risk factors are used to get the net return under the scenario
- The assumptions
  - The manager can represent his views as risk factor returns
  - To complete a scenario a stationary and normal multivariate distribution among all factors is assumed

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Inflation Linked Swaps

Inflation linked swaps involve a single payment at a specified maturity date when one party receives the swap notional compounded in line with actual inflation experience between inception and maturity in return for paying the notional compounded at a predefined growth rate defined in the contract

Valuation of an Option Exercisable Immediately

- For American options exercisable immediately, the option must be worth its intrinsic value

    - Call option: max ($S - K$, 0)

    - Put option: max ($K - S$, 0)

- For European options

    - Call option: max ($S\, e^{-q(T-t)} - K\, e^{-r(T-t)}$, 0)

    - Put option: max ($K\, e^{-r(T-t)} - S\, e^{-q(T-t)}$, 0)

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Valuation of Symmetric Derivatives

- In the absence of market frictions, symmetric derivatives can be valued by reference only to the principle of no arbitrage as long as suitable market observables exist to price the underlying building blocks.

- However, the valuation of asymmetric derivatives requires further assumptions which can be encapsulated in the risk neutral probability distribution for future outcomes

Single Step Binomial Tree

- This is a binomial lattice in which the underlying can only move in one of two ways at each discrete time step

- Assume that between each time step $t - h$ and t the price of the underlying can move from s to Su or Sd where $d < u$ and the potential movements are both nonnegative

- The value of the hedge portfolio is

  $$V(S, t - h) = p_u\, e^{-rh}\, V(Su, t) + p_d e^{-rh}\, V(Sd, t)$$

  Where

  $p_u = (e^{(\ -q))h} - d) / (u - d)$
  $p_d = (u - e^{(\ -q))h}) / (u - d)$
  $p_u + p_d = 1$

Multi-Step Binomial Tree

- In the first period, the underlying can move up or down by u or d and can move by a further u or d in later periods

- The price n periods back $t = T - nh$ with payoff at time T and u, d, p_u, p_d, r, q are the same for all periods

$$V(S, t - nh) = e^{-rnh} \sum_{m=0}^{n} \binom{n}{m} p_u^m \, p_d^{n-m} V(Su^m d^{n-m}, T)$$

Where $\binom{n}{m} = \frac{n!}{m!(n-m)!}$

- It can also be expressed as an expectation under a risk neutral probability distribution

$$V(S, t) = \mathrm{E}\left(e^{-r(T-t)} V(S, T) \mid S_t\right)$$

Tranches

Tranching is rearrangement of “who suffers what” if there are credit losses within a portfolio

Mechanics

- The lowest priority tranche (equity tranche) bears the first losses, then other tranches incur losses sequentially up the priority ladder
- The default frequency is not as important as the degree the observed default frequency exceeds the default rate implied by credit spreads within the CDO
- The attachment point is the level of loss which results in the tranche being repaid in full at maturity
- The detachment point is the level of loss at which the tranche receives nothing at maturity

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Reasons to Supplement VaR with Stress Testing

- Liquidity risk is major concern and not measured in VaR

- Portfolios are not well diversified because many risks are nonlinear or skewed

- Basic idea is to identify a range of scenarios which in the aggregate describe what might go wrong