

E-Learning Module Readings

Volume 2: Fundamentals

First Edition

Copyright © 2026
Society of Actuaries

All rights reserved under the U.S. and international copyright laws. No part of this work may be reproduced or distributed in any form without the express written permission of the Society of Actuaries.

Cover design by Rachel Maragnano

Product of the United States of America

About e-Learning Module Readings

This publication is a curated collection of readings designed to accompany and enrich your e-Learning experience. Unlike a traditional textbook, it does not follow a linear narrative. Instead, each article, excerpt, or transcript is intended to be engaged with as prompted from the e-Learning modules, reading-by-reading, as needed.

A Note on the Readings

The readings here are drawn from diverse sources: excerpts from published books, articles from a variety of publications, pieces written specifically for the modules, and transcripts from conferences or meetings. Because of this, there are considerable shifts in tone, style, and perspective across the readings. Some are formal and analytical; others are more conversational or practical. Even others are word-for-word transcripts of panel discussions. Maintaining this variety is intentional as it reflects the richness of the subject matter and the many voices contributing to the actuarial profession over the years. The original language of each source has been preserved to maintain its authenticity, even where that means stylistic differences across the readings.

How to Use this Publication

To get the most out of this collection:

If using a digital copy:

- **Read alongside the modules.** Keep the reader open in a separate window (or split-screen) while working through the e-Learning modules.
- **Search with purpose.** Use keyword search to quickly locate terms, authors, or concepts referenced in the module.
- **Highlight and save.** Take advantage of digital highlighting and note tools to flag key passages.

- **Bookmark your way throughout.** Create a personal set of bookmarks or saved locations so you can return to important readings instantly.

If using a printed copy:

- **Bookmark strategically.** Mark readings you expect to revisit, whether for professional reference or future work.
- **Annotate freely.** Jot notes in the margins to connect ideas across readings or back to the e-Learning content.
- **Create your own index.** Use tabs, sticky notes, or a simple system of markings to build quick access to the sections you rely on most.
- **Treat it as a working reference.** This volume is meant to be handled, revisited, and used over time, not simply read once.

Think of this book as a reference toolkit, not a cover-to-cover read. Dip into the readings as the module directs you, revisit them as needed, and let the interplay of perspectives deepen your understanding.

Acknowledgment of Contributors

This volume reflects decades of collective effort by practicing actuaries dedicated to the ongoing development of future professionals. The diversity of perspectives contained within these pages stands as testament to the collaborative nature of our profession. The Society of Actuaries (SOA) is profoundly grateful for their contributions.

Acknowledgment of Authors and Professional Sources

Countless published authors and professional actuaries have generously shared their knowledge and insights to the SOA's e-Learning modules over the years. Their contributions through original works, adapted materials, and subject matter expertise have been indispensable in creating this comprehensive resource.

Acknowledgment of Contributing Authors

This volume also includes original writings developed specifically for this publication. The individuals acknowledged below contributed material prepared expressly for inclusion here.

- **Karen L. Anway, FSA, FCA, EA**
- **Jeffrey A. Beckley, FSA, MAAA**
- **Stuart A. Klugman, FSA, CERA, PhD**
- **Matt Varitek, FSA, MAAA**
- **Toby A. White, FSA, CFA, PhD**

Brief biographies of these contributing authors appear at the end of the volume.

Contents

| | |
|-----------------------------------------------------------------------|-----|
| About e-Learning Module Readings | iii |
| Acknowledgment of Contributors | v |
| Risk in Actuarial Problems Module Readings | 1 |
| The Iceberg Principle | 3 |
| Recapping the Control Cycle | 6 |
| Comparing S&L Problems to Actuarial Problems | 9 |
| Editorial: Actuarial Aspects of the Recent Financial Market Crisis | 11 |
| Overview of Risks Related to Financial Security Systems | 14 |
| Stories from the Field | 27 |
| Scenario: Slick Oil Company | 34 |
| Risks of Actuarial Interest | 37 |
| The Financial Life Cycle | 39 |

| | |
|------------------------------------------------------------------------------|-----|
| Risks and Volatility in the Life Cycle | 42 |
| Perspectives on Retirement Risks and the Individual | 46 |
| Asset and Liability Management Risk Measures for Life Insurance Companies | 71 |
| Project Portfolio Selection: The Efficient-Frontier Approach | 76 |
| Interest Theory | 79 |
| Asset and Liability Management | 103 |
| Interest Rates | 120 |
| Introduction to Capital | 134 |
| Commonalities in Actuarial Problems | 140 |
| Ask an Actuary: Common Actuarial Concepts | 144 |
| Economics of Risk | 149 |
| Random Variables and Multidimensional Cash Flows | 151 |
| Economics of Risk: Revisited | 157 |
| The Time Value of Money | 159 |
| Individual and Collective Models | 165 |
| Introduction to Models and Modeling to Manage Risk | 169 |
| Jargon Busting: Communicating Without Creating Barriers | 178 |
| Everyday Applications of ASOP 41 | 182 |

| | |
|-----------------------------------------------------------------------------|------------|
| ASOP 41—The Do’s and Don’ts of Effective Actuarial Communication | 187 |
| Designing and Pricing an Actuarial Solution Module Readings | 191 |
| From the Field: Understanding Actuarial Models | 193 |
| Fundamental Actuarial Formulas: Pricing, Reserving, and Funding | 195 |
| Overview of Pricing Models | 200 |
| Pricing | 204 |
| Product Design | 210 |
| The Final Steps | 227 |
| Profit Measures | 229 |
| Analysis of Book Profits | 231 |
| Asset Shares | 258 |
| Sources of Profit | 267 |
| Estimating Claims Costs | 271 |
| Calculating Manual Gross Premiums | 277 |
| Calculating Benefit Factors | 283 |
| Experience Rating | 286 |
| Overview of Reserving Models | 293 |

| | |
|--------------------------------------------------------------------------|------------|
| Financial Reports: Liabilities on the Balance Sheet | 300 |
| Reserving | 307 |
| Short-Term Reserves | 312 |
| Long-Term Reserves | 319 |
| Reserves | 328 |
| Retirement Plan Solvency | 365 |
| US Pension Plan Design | 367 |
| More about Reserving Models | 372 |
| Overview of Funding Models | 378 |
| Overview of Employer-Sponsored Defined Benefit Plan Funding | 381 |
| Actuarial Assumptions Used in Defined-Benefit Plan Valuations | 385 |
| Benefit Allocation Methods | 389 |
| Cost Allocation Methods | 392 |
| Overview of Sensitivity Models | 394 |
| Sellers of Group Insurance Products | 397 |
| Cash Flow Projection Models | 405 |
| Behavioral Finance Models | 408 |
| Ask the Expert: Eric Clapprod | 411 |

| | |
|---------------------------------------------------------------------------------------------------------------------|------------|
| Model Selection and Solution Design Module Readings | 415 |
| Macro Pricing | 417 |
| Policy Reason for US Pension Accounting Requirements | 419 |
| Ask the Actuary: Contingent Tail Expectation (CTE) | 427 |
| Selection of Initial Assumptions Module Readings | 433 |
| Appropriate and Adequate Assumptions | 435 |
| Known and Unknown Knowns and Assumptions | 437 |
| Ask an Actuary: Viatical Settlements | 441 |
| Selection of Initial Assumptions within the Control Cycle | 443 |
| The Process of Selecting Assumptions | 446 |
| Ask an Actuary: Interest Rate Assumptions Used in Enterprise Risk Management and Pension Plan Valuations | 453 |
| Keeping Pace with Social and Demographic Changes of an Aging Population ... You Have to Be Fast on Your Feet | 456 |
| Choosing Assumptions | 464 |
| Ask an Actuary: Should Assumptions Be Reasonable? | 465 |
| Let's Talk: Discussion Is Needed on Actuarial Assumptions, Consistency | 466 |

| | |
|-----------------------------------------------------------------|-----|
| Assumption Criteria | 469 |
| Considerations for Setting Assumptions | 471 |
| Pricing Challenges Today | 475 |
| Modifying Data | 499 |
| Building Block Assumptions: An Example | 501 |
| Scenario Generation: Valuation versus Strategy Development | 502 |
| Materiality | 507 |
| Assumptions Techniques | 510 |
| Getting to Know CTE | 514 |
| Investing / Portfolio Management | 525 |
| Finance Models and Data | 547 |
| Additional Thoughts on Developing Assumptions | 558 |
| The Perils of Headstrong Health Insurance Company | 560 |
| Valuation Actuary Overview: The New Statutory Valuation Opinion | 566 |
| Applied Futurism | 591 |
| Monitoring Results Module Readings | 603 |
| Monitoring Results: What It Really Means in Practice | 605 |
| Analysis of Profit or Loss | 608 |

| | |
|--------------------------------------------------------------------------------------------------------------------------|------------|
| Expenses | 614 |
| Experience Rating | 617 |
| Bucking Tradition: A Roundtable Interview with Three Actuaries Who Have Gone Down Unexpected Career Paths | 623 |
| Contributor Biographies | 631 |



Society of
Actuaries®

E-Learning Module Readings

Volume 2: Fundamentals

First Edition

FUNDAMENTALS OF ACTUARIAL PRACTICE

Risk in Actuarial Problems
Module

**Designing and Pricing an
Actuarial Solution Module**

Model Selection and
Solution Design Module

Selection of Initial
Assumptions Module

Monitoring Results Module

Fundamental Actuarial Formulas: Pricing, Reserving, and Funding

In this section, we describe the fundamental actuarial formula used for pricing, reserving, and funding decisions.

For the fundamental actuarial formula, this we use the following simplified formula:

$$V = \sum B_t d_t v^t - \pi_t \Delta_t v^t$$

For our purposes, “primary” models are models intended to solve problems associated with pricing, reserving, or funding financial security systems. The three primary models described in this reading provide the base of the model framework. The key questions addressed by each model are:

- **Pricing Model:** Is there a balance between the premiums and the benefits?
- **Reserving Model:** Are current funds sufficient to meet the financial security system’s future needs?
- **Funding Model:** Will future funds be sufficient to meet the financial security system’s future needs?

Note: The terms “primary models” and “secondary models” are convenient, though not generally used, terms employed to distinguish their purposes. Typically, the simpler term “models” is used when referring to any of the parameter—and assumption—setting tools that represent observed, measurable, real-world phenomena.

The Pricing Formula

When pricing, the key question to ask is whether there is a balance between the premiums and the combined benefits, expenses, and profits. In other words, does initial income equal outgo? Are the premiums

reasonable, considering the obligations of the financial security system? And are the benefits reasonable, given the premiums available?

With these questions as a backdrop, (simplified) pricing is essentially a point estimate as of today. The pricing formula, then, can be shown as

$$0 = V_0 = \sum_{t=0}^{\infty} B_t d_t v^t - \pi_t \Delta_t v^t$$

where:

- V_0 represents assets at inception, which equals zero.
- $B_t d_t v^t$ represents the value at inception of benefits (including expenses and profits) payable in year t .
- $\pi_t \Delta_t v^t$ represents the value at inception of premiums payable in year t .
- v^t represents the discount for interest from time t .
- d_t represents the adjustment for all other decrements to time t with respect to benefit cash flows.
- Δ_t represents the adjustment for all other decrements to time t with respect to premium cash flows.

It is important to note that the Fundamentals of Actuarial Practice uses the words “premiums” and “benefits” in a generic sense. Benefits include all payments out of the financial security system, and premiums include all payments into the financial security system. In a pension plan, for example, premiums would include contributions made into a pension fund.

In the health practice area, pricing would be used to calculate gross premiums. These premiums are based on claims costs, expenses, premiums, and other taxes, commissions, risk and profit charges, and adjustments for investment income. All of these factors are represented in the pricing formula.

It should be apparent from the pricing formula that the goal for pricing is to set the value of expected income at inception, to the value of expected outgo at inception. This is the basis for pricing under any financial security system.

The Reserving Formula

When reserving, the key question to ask is whether current funds are sufficient to meet the financial security system's future needs.

The reserving formula can be shown as:

$$V_x = \sum_{t=x}^{\infty} B_t d_{x,t} v^{t-x} - \pi_t \Delta_{x,t} v^{t-x}$$

where:

- V_x represents the assets required in the future.
- $B_t d_{x,t} v^{t-x}$ represents the value at x of benefits payable in year t .
- $\pi_t d_{x,t} v^{t-x}$ represents the value at x of premiums payable in year t .
- v^{t-x} represents the discount for interest from time t to time x .
- $d_{x,t}$ represents the adjustment for all other decrements from x to time t with respect to benefit cash flows.
- $\Delta_{x,t}$ represents the adjustment for all other decrements to time t with respect to premium cash flows.

The reserving formula can also be depicted as follows:

$$V_x = \sum_{t=0}^{x-1} \pi_t d_{t,x} v^{t-x} - B_t \Delta_{t,x} v^{t-x}$$

This is the *retrospective* formula, rather than the prospective formula.

Reserving, then, results in a point estimate after the inception of the financial security system. For example, you would use the reserving formula after the introduction of a particular insurance product or after the introduction of a retirement benefits program.

Don't be fooled by the fact that some reserving formulas (and the models upon which they are based) can be written in the following form:

$$V_x = \sum_{t=x}^{\infty} B_t d_{x,t} v^{t-x}$$

In this case, the future premiums (income) are nil. This is a very common reserving formula for claims that are in play.

Note that the pricing formula is, in fact, the same as the reserving formula applied at time 0. It must be remembered, however, that the underlying

formula is only part of a model. The assumptions used to determine benefits, premiums, and decrements in reserving are not necessarily the same as those used for pricing. The amounts used for each of the factors—the B_t , the d_t , the π_t , and the interest rate underlying the v —must all be specifically chosen for each problem at hand. For example, an actuary calculating statutory reserves for a life insurance product will use premiums and reserving assumptions that are specific to that task and depend on the requirements in the actuary’s specific jurisdiction.

The Funding Formula

When funding, the key question to ask is whether future funds will be sufficient to meet the financial security system’s future needs. Funding is a dynamic process that requires the successive application of the reserving formula.

Funding can be considered a point estimate for all points in time and can be shown as follows:

$$V_x = \sum_{t=x}^{\infty} B_t d_{x,t} v^{t-x} - \pi_t \Delta_{x,t} v^{t-x}, \quad x \geq 0$$

where:

- V_x represents the assets required in the future at time x .
- $B_t d_{x,t} v^{t-x}$ represents the value at x of benefits payable in year t .
- $\pi_t d_{x,t} v^{t-x}$ represents the value at x of premiums payable in year t .
- v^{t-x} represents the discount for interest from time t to time x .
- $d_{x,t}$ represents the adjustment for all other decrements from x to time t with respect to benefit cash flows.
- $\Delta_{x,t}$ represents the adjustment for all other decrements to time t with respect to premium cash flows.
- $x \geq 0$ illustrates that the formulas apply at all points in the future.

Secondary Models

The parameters for the primary models often have to be modeled themselves. For example, B_t at each time t is often modeled separately in a secondary model, and the results from the secondary model are then used as input into the primary model. In this book, the models or techniques used for these purposes are called secondary models.

Actuaries refer to experience studies or experience rating techniques as models. They are models in that they help you represent observed, measurable, real-world phenomena. In our terminology, they are generally secondary models.

Secondary models support primary models by specifying a possible parameter (for example, B_t) for the primary model. Experience studies, experience rating, and sensitivity models can be used to support the pricing, reserving, or funding calculations. For example, experience rating uses a recursive formula to solve for current benefits, using previous benefits and a prior expected current benefit.



Society of
Actuaries®

E-Learning Module Readings

Volume 2: Fundamentals

First Edition

FUNDAMENTALS OF ACTUARIAL PRACTICE

Risk in Actuarial Problems
Module

Designing and Pricing an
Actuarial Solution Module

Model Selection and
Solution Design Module

**Selection of Initial
Assumptions Module**

Monitoring Results Module

Known and Unknown Knowns and Assumptions

ontology, on-tol-uh-jee, noun

*The branch of metaphysics dealing with the philosophical theory of reality, including consideration of the universal and necessary characteristics of all existence; also a particular theory of reality.*¹

Former US Secretary of Defense Donald M. Rumsfeld, at a 2002 Department of Defense news briefing on Iraq, made the following statement about the knowledge available:

Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns—the ones we don't know we don't know.

Most of us are quite able to define the part of knowledge we know. We are less confident when stating how much “I know I know.” We are also quite confident when stating how much “I know I don't know.” We typically go through life happy in the knowledge that there are things we do know and things we don't know. When asked, however, to contemplate the part of knowledge we have not even considered—the “I don't know I don't know” part—most of us are at first confused and then humbled. The part of knowledge we don't know we don't know is huge.

Try answering these questions:

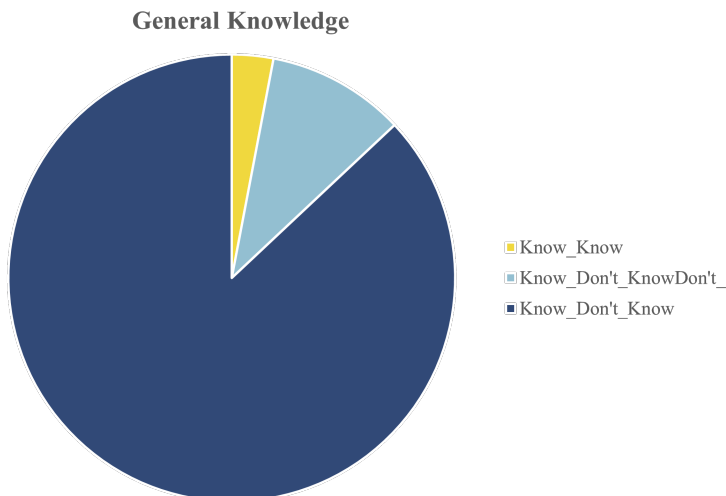
- What proportion of the total potential knowledge pool would you characterize as knowledge “I know I know”?

¹*Funk and Wagnalls Standard College Dictionary*. New York: Funk and Wagnalls Publishing Company, Inc., 1974. s.v. “ontology.” Accessed September 2, 2025. https://archive.org/details/funkwagnallsstan0000unse_u7t2/page/944/mode/2up.

- What proportion would you characterize as knowledge “I know I don’t know”?
- What proportion would you characterize as knowledge “I don’t know I don’t know”?

In general, what a person doesn’t even know they don’t know represents by far the biggest share of all potential knowledge. Most people never even consider that portion of the knowledge pool. Figure 1 depicts a rough representation of the average knowledgeable person’s “typical” answer to the three questions.²

Figure 1: General Knowledge: Typical Perceptions



As an actuary, you should aim to increase the amount of relevant knowledge that you know you know.

It should also be your goal to change, through research and training, some of the “I know I don’t know” knowledge to the “I know I know” category.

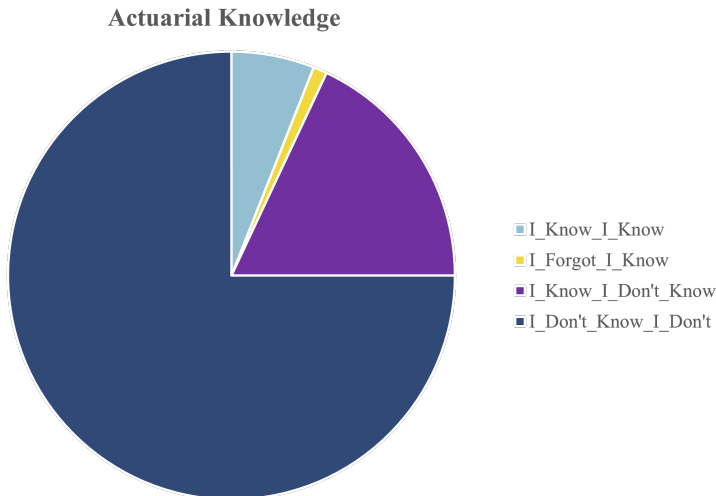
A third important goal is to change some of the “I don’t know I don’t know” knowledge to one of the other two categories. Some ways to accomplish this are to be aware and look for new indicators of knowledge.

²IBM Distinguished Professor of Regulation and Competitive Practice & Director of Ford Center for Global Citizenship; Co-Director, Northwestern Institute on Complex Systems (NICO), Kellogg School of Management (MEDS), Northwestern University

A mathematician wants to consider all the permutations and combinations, so you should also consider a fourth category of knowledge about knowledge: “I don’t know, I know.” It can be argued that a person can’t not know what they know, which would make this fourth condition a subset of the “I know I know” condition. However, this fourth combination cannot be ignored in actuarial work, and a clearer way to express it might be “I forgot that I know.”

This fourth combination can be very important, as you do not want to inadvertently ignore any readily accessible information when you are dealing with actuarial problems. The knowledge graphic of an effective professional, with regard to the actuarial knowledge pool, should therefore look more like Figure 2.

Figure 2: Actuarial Knowledge: A Professional’s Perspective



These categories of knowledge suggest criteria for the actuarial task of selecting assumptions: Know what you know. Make sure you don’t forget what you know. Consider what you know you don’t know. And be aware that you must consider what you don’t know you don’t know.

Most actuarial problems are focused on future contingent events, or events dependent upon an uncertain condition—the unknown. To find the optimal solution, the actuary must make certain assumptions with respect to the uncertain future conditions that will affect the contingent events. In selecting the assumptions, the actuary must recognize known known items, such as historical experience and current data. The actuary must be careful not to overlook information that he or she is aware of

but may not be thinking of in the context of the problem (the unknown knowns). Then, the actuary must make sure the assumption-setting process recognizes and provides for future changes to historical patterns and current data (the known unknowns). Finally, the actuary considers the possibility of future occurrences of events that have never occurred in the past (the unknown unknowns).

In selecting assumptions, an actuary wants to ensure that the known known items are accurate and properly reflected in his or her models. Care should be taken not to ignore readily accessible information or knowledge that may be relevant to the problem. The actuary also wants to be able to illustrate and quantify the effect of various assumptions for known unknown current and future conditions (i.e., specific scenarios). While assumptions also often provide for the possibility of future unforeseen corrections, the monitoring of results helps the actuary identify variances of actual results from expected results, due to previously unforeseen contingencies (unknown unknowns).

The ability to deal with the unknown makes actuarial science challenging, fun, and dependent on realistic, quantifiable, and variable assumptions that must be monitored throughout the stages of the Actuarial Control Cycle: Define the Problem, Design the Solution, and Monitor the Results.

Contributor Biographies

Karen L. Anway, FSA, FCA, EA

Karen retired at the end of 2023 after 33 years with WTW, where she served as Director of Operations for the North America Retirement Business and Senior Consulting Actuary, providing actuarial consulting services for clients' retirement and healthcare programs. Karen graduated from GMI (now Kettering University) and has volunteered for the Society of Actuaries since 2002. Among other roles, she has served on the Board of Directors, the Education Executive Committee, the Retirement Curriculum Committee, the Retirement Examination Committee, and the ASA and FSA Redesign Task Force.

Jeffrey A. Beckley, FSA, MAAA

Jeff retired at the end of 2022 after 18 years at Purdue University. He served as the Director of the Actuarial Science Program for his last 10 years at Purdue. Prior to joining Purdue in 2003, he spent 18 years as a consulting actuary for life and health insurance companies, as well as seven years working for life insurance companies. Jeff is a 1978 graduate of Ball State University. He has been a consistent volunteer for the Society of Actuaries since earning his Fellowship in 1980. Among other roles, he has served as the Examination Chairperson, the Chair of the Committee on Life Insurance Research, and Chair of the Executive Research Committee. Jeff has served as a member of the Board of Directors for the Society of Actuaries and has acted as a Facilitator for the Fellowship Admissions Course (FAC) multiple times since 2012. Jeff received the Presidential Award for Service to the SOA in 2001 and 2021. He continues to be an active volunteer for the SOA as well as an avid gardener and runner.

Stuart A. Klugman, FSA, CERA, PhD

Stuart is a Senior Staff Fellow in Education at the Society of Actuaries (SOA), a position he has held since 2009. Prior to that, he was Professor

of Actuarial Science at The University of Iowa (1974–1988) and Drake University (1988–2009). He earned a B.S. in actuarial science from Drake University in 1970 and a Ph.D. in statistics from the University of Minnesota in 1975. He served on the SOA Board as a member and as Vice President. He co-chaired the committee that redesigned the SOA's education system in 2005 and has been extensively involved in the creation and maintenance of the e-Learning modules. More recently, he led the SOA's expansion into predictive analytics at both the certificate and exam levels. He is a co-author of *Loss Models: From Data to Decisions*, a required text for Exams FAM and ASTAM. In 2007 and 2012, he received the SOA Presidential Award.

Matt Varitek, FSA, MAAA

Matt serves as an actuary for AHCCCS, the Arizona Medicaid plan. AHCCCS covers health care services for over 1.7 million Arizonans in need. Matt has served AHCCCS for over ten years, in addition to 17 years of experience in the private sector, primarily with BCBS plans in Wisconsin and Arizona. He earned an FSA in the SOA in 2009 and has since pursued various volunteer efforts. He is a 1996 graduate of Illinois State University. He is a past member of the SOA Education Executive Committee, a past chair of the FAP Interim Assessment Grading Team, and a past chair of the FAP Curriculum Committee. In 2013, he was recognized as an Outstanding Volunteer by the SOA at the Annual Meeting.

Toby A. White, FSA, CFA, PhD

Toby is an Associate Professor of Actuarial Science and Finance at Drake University, where he has taught full-time since 2008. After graduating from Kalamazoo College and the University of Iowa, Toby worked for four years at The Lincoln Financial Group in the mid-to-late 1990s. Thereafter, Toby served as a consultant for Tillinghast-Towers Perrin in Chicago and a visiting lecturer at the University of Illinois in Champaign-Urbana. Toby became a Fellow of the Society of Actuaries in 2000 and earned the Chartered Financial Analyst (CFA) designation in 2003. In 2008, he earned a Ph.D. in Statistics from the University of Washington in Seattle. He currently writes questions for Exams P, FM, and FAM and has served on committees for Courses 5, 6, 7, Exam MFE/IFM, and the FAP Modules in the past. Before joining the MFE exam committee, Toby also authored a study guide for the MFE exam. He has recently been involved in curriculum redesign initiatives related to Exams P and FM, as well as the entire ASA portfolio of exams, dating back to the

2010s. He has been the Accreditation Actuary at Drake University since 2022, overseeing the internal operations of Drake's UEC program.